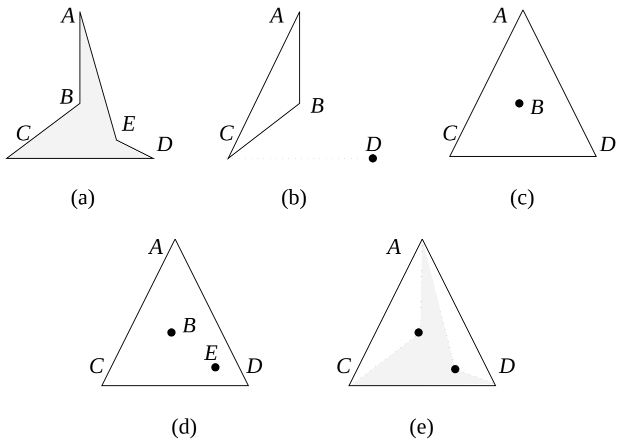


Figure 8.30: Example of convex hull detection. (a) The processed region—polygon $ABCDEA$. (b) Vertex D is entered and processed. (c) Vertex D becomes a new vertex of the current convex hull ADC. (d) Vertex E is entered and processed, E does not become a new vertex of the current convex hull. (e) The resulting convex hull $DCAD$.

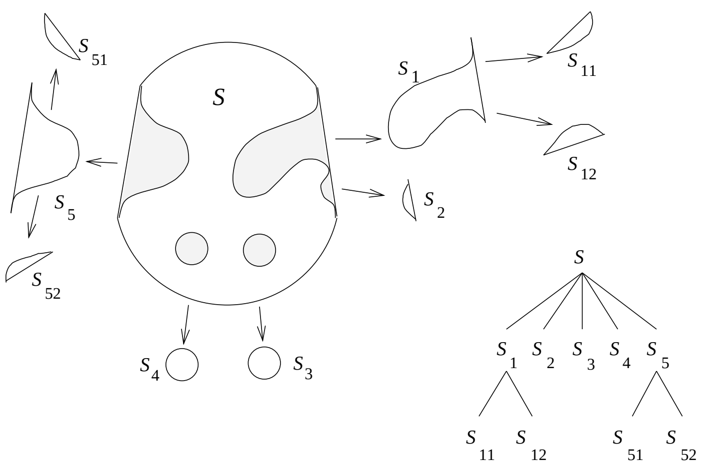


Figure 8.31: Concavity tree construction. (a) Convex hull and concave residua. (b) Concavity tree.

8.3.4 Graph representation based on region skeleton

This method corresponds significantly curving points of a region boundary (Section 8.2.2) to graph nodes. The main disadvantage of boundary-based description methods is that geometrically close points can be far away from one another when the boundary is described—graphical representation methods overcome this disadvantage. Shape properties are then derived from the graph properties.

The region graph is based on the region skeleton, and the first step is the skeleton construction. There are four basic approaches to skeleton construction:

- Thinning—iterative removal of region boundary pixels.
- Wave propagation from the boundary.

- Detection of local maxima in the distance-transformed image of the region.
- Analytical methods.

Expected properties of skeletonization algorithms include [Bernard and Manzanera, 1999]:

- Homotopy – skeletons must preserve the topology of the original shapes/images.
- One-pixel thickness – skeletons should be made of one-pixel thick lines.
- Mediality – skeletons should be positioned in the middle of shapes (with all skeleton points having the same distance from two closest points on object boundary).
- Rotation invariance – in discrete spaces, this can only be satisfied for rotation angles, which are multiples of $\pi/2$, but should be approximately satisfied for other angles.
- Noise immunity – skeletons should be insensitive to shape-boundary noise.

Some of these requirements are contradictory—noise immunity and mediality cannot be satisfied simultaneously. Similarly, rotation invariance and one-pixel thickness requirements work against each other. While all five requirements contribute to the quality of resulting skeletons, satisfying homotopy, mediality, and rotation invariance is of major importance [Manzanera et al., 1999].

Most thinning procedures repeatedly remove boundary elements until a pixel set with maximum thickness of 1 or 2 is found. In general, these methods can be either sequential, iteratively directionally parallel, or iteratively fully parallel. The following MB algorithm is an iteratively fully parallel skeletonization algorithm and it constructs a skeleton of maximum thickness 2 [Manzanera et al., 1999]. It is simple, preserves topology (i.e., no single component is deleted or split into several components, no object cavity is merged with the background or another cavity, and no new cavity is created) and it is geometrically correct (i.e., objects are shrunk uniformly in all directions and the produced skeleton lines are positioned in the middle of the objects). While it has limited rotational invariance, it is computationally fast.

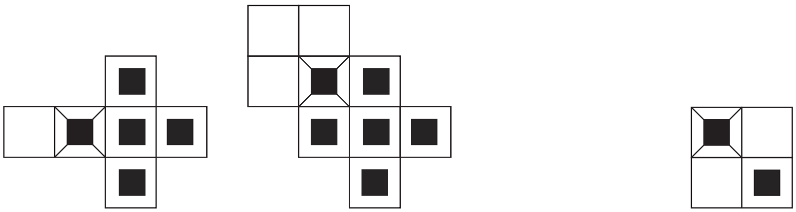
Algorithm 8.9: Fully parallel skeleton by thinning – MB algorithm

1. Consider a binary image consisting of object pixels and background pixels.
2. Identify a set $\mathcal{Y}$ of object pixels, for which the thinning mask shown in Figure 8.32a matches the local image configuration while the restoring mask Figure 8.32b does not match the local image configuration. This step is performed in parallel for all object pixels of the image and all $\pi/2$ rotations of the masks.
3. Remove all object pixels $\mathcal{Y}$.
4. Repeat the two previous steps as long as $\mathcal{Y}$ is nonempty.

A refined version of the MB skeletonization algorithm—called MB2—offers substantially improved rotational invariance while maintaining all other good properties [Bernard and Manzanera, 1999]. While still computationally fast when compared to other approaches, it is somewhat slower than Algorithm 8.9.

(a) (b)

Figure 8.32: Masks for the MB skeletonization algorithm [Manzanera et al., 1999]. We include all 90-degree rotations of these two. Panel (a) shows the thinning mask (plus all $\pi/2$ rotations). Panel (b) shows the restoring mask (plus all $\pi/2$ rotations). The central mask pixel is marked with a diagonal cross, background pixels are white and object pixels are black.



(a) (b) (c)

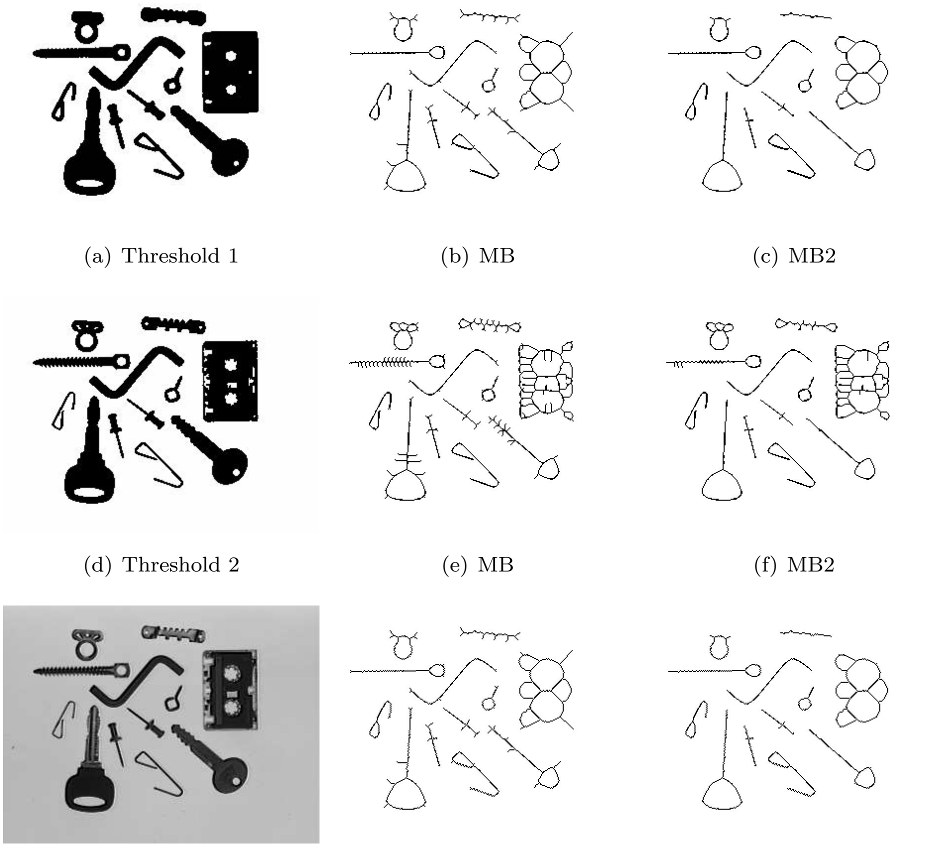
Figure 8.33: Masks for the MB2 skeletonization algorithm [Bernard and Manzanera, 1999]—we include all $\pi/2$ rotations of them. Panels (a) and (b) show the thinning masks (plus all $\pi/2$ rotations). Panel (c) shows the restoring mask (plus all $\pi/2$ rotations). The central mask pixel is marked with a diagonal cross, background pixels are white and object pixels are black.

Algorithm 8.10: Fully parallel skeleton by thinning – MB2 algorithm

1. Consider a binary image consisting of object pixels and background pixels.
2. Identify a set $\mathcal{Y}$ of object pixels, for which at least one of the thinning masks shown in Figure 8.33a,b matches the local image configuration while the restoring mask Figure 8.33c does not. This step is performed in parallel for all object pixels of the image.
3. Remove all object pixels $\mathcal{Y}$.
4. Repeat the two previous steps as long as $\mathcal{Y}$ is nonempty.

Examples of MB and MB2 skeletons and the effects on them resulting from minor changes of object shapes due to variations in segmentation threshold can be see in Figure 8.34.

Since the MB and MB2 algorithms yield skeleton segments which may have a thickness of 1 or 2, (Figure 8.34b,e) an extra step can be added to reduce those to a thickness of one, although care must be taken not to break the skeleton connectivity. One-pixel skeleton thickness can be obtained using an asymmetric two-dimensional thinning algorithm as a post-processing step, in which simple points are removed [Rosenfeld, 1975]. While removal of a simple-point pixel will not alter topology, parallel removal of two or more of such pixels may result in a topology change. In other words, if all candidate



(a) Threshold 1 (b) MB (c) MB2

(d) Threshold 2 (e) MB (f) MB2

(g) Original (h) 1-pixel skeleton from (b) (i) 1-pixel skeleton from (c)

Figure 8.34: MB and MB2 skeletons from skeletonization of image in Figure **??**a (shown here in panel (g)). These skeletonization algorithms produce 1- or 2-pixel skeletons. (a) and (d) Binary images resulting from thresholding of panel (g). (b) and (e) MB skeletons. (c) and (f) MB2 skeletons. (g) Original image. (h) 1-pixel wide MB skeleton of image in panel (a)—derived from MB skeleton of panel (b). (i) 1-pixel wide MB2 skeleton of image in panel (a)—derived from MB2 skeleton of panel (c). Note the effect of different threshold on the resulting skeleton—compare panels (a–c) and (d–f). Courtesy Li Zhang, The University of Iowa.

pixels are removed in parallel, topology may be affected and the skeleton may break into pieces. The basic idea of obtaining a 1-pixel wide skeleton using this approach [Rosenfeld, 1975] is therefore to divide the thinning process in substeps and in each substep remove—in parallel—all pixels that have no neighbor belonging to the object in exactly one of the four main directions (north, south, east, west). The 4 directions are rotated in subsequent applications of the parallel pixel removal substeps. The substeps are repeated until convergence—as long as at least one pixel can be removed during the substep. This strategy results in a one-pixel wide skeleton while preserving its topology.

A large number of thinning algorithms can be found in the literature [Hildich, 1969; Pavlidis, 1978] and a useful comparison of parallel thinning algorithms is in [Couprie, 2005]. Mathematical morphology is another powerful tool used to find region skeletons, and thinning algorithms which use morphology are given in Section ??; see also [Maragos and Schafer, 1986], where the morphological approach is shown to unify many other approaches to skeletonization.

Thinning procedures often use a medial axis transform (also symmetric axis transform) to construct a region skeleton [Pavlidis, 1977; Samet, 1985; Pizer et al., 1987; Lam et al., 1992; Wright and Fallside, 1993]. Under the medial axis definition, the skeleton is the set of all region points which have the same minimum distance from the region boundary for at least two separate boundary points. Examples of such skeletons are shown in Figures 8.35 and 8.36. Such a skeleton can be constructed using a distance transform which assigns a value to each region pixel representing its (minimum) distance from the region's boundary, and the skeleton is then determined as the set of pixels whose distance from the region's border is locally maximal. As a post-processing step, local maxima can be detected using operators that detect linear features and roof profiles [Wright and Fallside, 1993]. Every skeleton element can be accompanied by information about its distance from the boundary—this gives the potential to reconstruct a region as an envelope curve of circles with center points at skeleton elements and radii corresponding to the stored distance values. Shape descriptions, as discussed in Section 8.3.1 can be derived from this skeleton but, with the exception of elongatedness, the evaluation can be difficult. In addition, this skeleton construction is time-consuming, and the result is highly sensitive to boundary noise and errors. Small changes in the boundary may cause serious changes in the skeleton—see Figure 8.35. This sensitivity can be removed by first representing the region as a polygon, then constructing the skeleton. Boundary noise removal can be absorbed into the polygon construction. A multi-resolution (scale-space) approach to skeleton construction may also result in decreased sensitivity to boundary noise [Pizer et al., 1987; Maragos, 1989]. Similarly, the approach using the Marr-Hildreth edge detector with varying smoothing parameter facilitates scale-based representation of the region's skeleton [Wright and Fallside, 1993].

Skeleton construction algorithms do not result directly in graphs, but the transformation from skeletons to graphs is relatively straightforward. Consider first a 1-pixel wide skeleton—this is advantageous since any skeleton pixel A with only one neighbor corresponds to a leaf vertex (end point) of the graph, pixels with 3 or more neighbors are associated with branching graph nodes (node points), and all remaining skeleton pixels with 2 neighbors (normal points) translate to arcs between branching and/or leaf vertices. Now consider medial axis skeletons and assume that a minimum radius circle has been drawn from each point of the skeleton which has at least one point common with a region boundary: let *contact* be each contiguous subset of the circle which is common to the circle and to the boundary. If a circle drawn from its center A has one contact only,

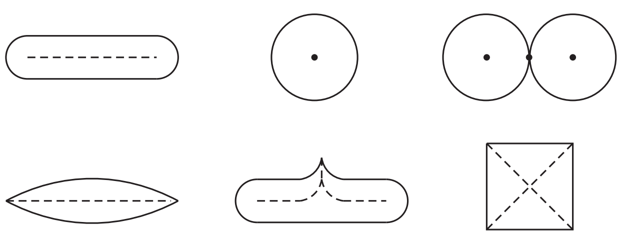
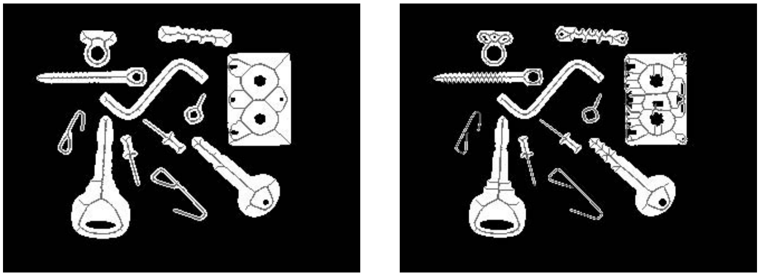


Figure 8.35: Region skeletons; small border changes can have a substantial effect on skeleton.



(a) (b)

Figure 8.36: Medial axis skeletons [Pavlidis, 1981] overlaid in mid-level gray over original binary data given in Figure 8.34a,d. Courtesy Kalman Palagyi, University of Szeged, Hungary.

A is a skeleton end point. If the point A has two contacts, it is a normal skeleton point. If A has three or more contacts, the point A is a skeleton node point.

Algorithm 8.11: Region graph construction from skeleton

1. Label each skeleton point as one of end point, node point, normal point.
2. Let graph node points be all end points and node points. Connect any two graph nodes by a graph arc (graph edge) if they are connected by a sequence of normal points in the region skeleton.

It can be seen that boundary points of high curvature have the main influence on the graph. They are represented by graph nodes, and therefore influence the graph structure.

If other than medial axis skeletons are used for graph construction, end points can be defined as skeleton points having just one skeleton neighbor, normal points as having two skeleton neighbors, and node points as having at least three skeleton neighbors. It is no longer true that node points are never neighbors and additional conditions must be used to decide when node points should and should not be represented as nodes in a graph.

8.3.5 Region decomposition

The decomposition approach is based on the idea that shape recognition is a hierarchical process. Shape **primitives**—the simplest elements which form the region—are defined at the lower level. A graph is constructed at the higher level—nodes result from primitives, arcs describe the mutual primitive relations. Convex sets of pixels are one example of simple shape primitives.

The solution to the decomposition problem consists of two main steps: The first step is to segment a region into simpler sub-regions (primitives), and the second is their analysis. Primitives are simple enough to be described successfully using simple scalar shape properties (see Section 8.3.1). A detailed description of how to segment a region into primary convex sub-regions, methods of decomposition to concave vertices, and graph construction resulting from a polygonal description of sub-regions are given in

(c) Explain the differences in performance of your algorithm.

(d) Develop a practically applicable thinning algorithm that constructs line shapes from scanned characters.

8.7 References

Ansari N. and Delp E. J. Distribution of a deforming triangle. *Pattern Recognition*, 23(12): 1333–1341, 1990.

Appel K. and Haken W. Every planar map is four colourable: Part I: discharging. *Illinois Journal of Mathematics*, 21:429–490, 1977.

Asada H. and Brady M. The curvature primal sketch. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 8(1):2–14, 1986.

Atkinson H. H., Gargantini, I, and Walsh T. R. S. Counting regions, holes and their nesting level in time proportional to the border. *Computer Vision, Graphics, and Image Processing*, 29:196–215, 1985.

Babaud J., Witkin A. P., Baudin M., and Duda R. O. Uniqueness of the Gaussian kernel for scale-space filtering. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 8 (1):26–33, 1986.

Ballard D. H. and Brown C. M. *Computer Vision*. Prentice-Hall, Englewood Cliffs, NJ, 1982.

Bay H., Ess A., Tuytelaars T., and Van Gool L. Speeded-Up Robust Features (SURF). *Computer Vision Image Understanding*, 110(3):346–359, 2006.

Beis J. and Lowe D. Shape indexing using approximate nearest-neighbour search in high-dimensional spaces. In *Proceedings of the Conference on Computer Vision and Pattern Recognition CVPR*, pages 1000–1006, Puerto Rico, 1997.

Bernard T. M. and Manzanera A. Improved low complexity fully parallel thinning algorithm. In *Proc. 10th International Conference on Image Analysis and Processing, ICIAP'99*, pages 215–220, 1999.

Bhanu B. and Faugeras O. D. Shape matching of two–dimensional objects. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6(2):137–155, 1984.

Bookstein F. L. *Morphometric Tools for Landmark Data*. Cambridge University Press, Cambridge, 1991.

Bribiesca E. and Guzman A. How to describe pure form and how to measure differences in shapes using shape numbers. *Pattern Recognition*, 12(2):101–112, 1980.

Cash G. L. and Hatamian M. Optical character recognition by the method of moments. *Computer Vision, Graphics, and Image Processing*, 39:291–310, 1987.

Chien C. H. and Aggarwal J. K. Model construction and shape recognition from occluding contours. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(4):372–389, 1989.

Cinque L. and Lombardi L. Shape description and recognition by a multiresolution approach. *Image and Vision Computing*, 13:599–607, 1995.

Coeurjolly D., M S., and Tougne L. Discrete curvature based on osculating circle estimation. In *Proceedings of the 4th International Workshop on Visual Form*, IWVF-4, pages 303–312, London, UK, 2001. Springer-Verlag.

Cootes T. F., Cooper D. H., Taylor C. J., and Graham J. Trainable method of parametric shape description. *Image and Vision Computing*, 10(5), 1992.

Cortopassi P. P. and Rearick T. C. Computationally efficient algorithm for shape decomposition. In *CVPR '88: Computer Society Conference on Computer Vision and Pattern Recognition,* Ann Arbor, MI, pages 597–601, Los Alamitos, CA, 1988. IEEE.

Costabile M. F., Guerra C., and Pieroni G. G. Matching shapes: A case study in time-varying images. *Computer Vision, Graphics, and Image Processing*, 29:296–310, 1985.

Couprie M. Note on fifteen 2d parallel thinning algorithms. Technical Report IGM2006-01, Universite de Marne-la-Vallee, 2005.

Debled-Rennesson I. and Reveillès J. P. A linear algorithm for segmentation of digital curves. *IJPRAI*, 9:635–662, 1995.

DeBoor C. A. *A Practical Guide to Splines*. Springer Verlag, New York, 1978.

Duda R. O. and Hart P. E. *Pattern Classification and Scene Analysis.* Wiley, New York, 1973.

Dyer C. R. Computing the Euler number of an image from its quadtree. *Computer Graphics and Image Processing*, 13:270–276, 1980.

Fermuller C. and Kropatsch W. Multi-resolution shape description by corners. In *Proceedings, 1992 Computer Vision and Pattern Recognition,* Champaign, IL, pages 271–276, Los Alamitos, CA, 1992. IEEE.

Fisher D. J., Ehrhardt J. C., and Collins S. M. Automated detection of noninvasive magnetic resonance markers. In *Computers in Cardiology,* Chicago, IL, pages 493–496, Los Alamitos, CA, 1991. IEEE.

Flusser J. and Suk T. Pattern recognition by affine moment invariants. *Pattern Recognition*, 26:167–174, 1993.

Forsyth D., Mundy J. L., Zisserman A., Coelho C., Heller A., and Rothwell C. Invariant descriptors for 3D object recognition and pose. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(10):971–991, 1991.

Freeman H. On the encoding of arbitrary geometric configuration. *IRE Transactions on Electronic Computers*, EC–10(2):260–268, 1961.

Fu K. S. *Syntactic Methods in Pattern Recognition.* Academic Press, New York, 1974.

Gross A. and Latecki L. Digital geometric invariance and shape representation. In *Proceedings of the International Symposium on Computer Vision*, pages 121–126, Los Alamitos, CA, 1995. IEEE.

Gupta L. and Srinath M. D. Contour sequence moments for the classification of closed planar shapes. *Pattern Recognition*, 20(3):267–272, 1987.

Gupta L., Sayeh M. R., and Tammana R. Neural network approach to robust shape classification. *Pattern Recognition*, 23(6):563–568, 1990.

Held A. and Abe K. On the decomposition of binary shapes into meaningful parts. *Pattern Recognition*, 27:637–647, 1994.

Hermann S. and Klette R. Multigrid analysis of curvature estimators. Technical Report CITR-TR-129, Massey University, 2003.

Hermann S. and Klette R. A comparative study on 2d curvature estimators. In *ICCTA*, pages 584–589. IEEE Computer Society, 2007.

Hildich C. J. Linear skeletons from square cupboards. In Meltzer B. and Michie D., editors, *Machine Intelligence IV*, pages 403–420. Elsevier, New York, 1969.

Hogg D. C. Shape in machine vision. *Image and Vision Computing*, 11:309–316, 1993.

Hu M. K. Visual pattern recognition by moment invariants. *IRE Transactions Information Theory*, 8(2):179–187, 1962.

Jain A. K. *Fundamentals of Digital Image Processing.* Prentice-Hall, Englewood Cliffs, NJ, 1989.

Jakubowski R. Decomposition of complex shapes for their structural recognition. *Information Sciences*, 50(1):35–71, 1990.

Jiang X. Y. and Bunke H. Simple and fast computation of moments. *Pattern Recognition*, 24: 801–806, 1991.

Juday R. D., editor. *Digital and Optical Shape Representation and Pattern Recognition,* Orlando, FL, Bellingham, WA, 1988. SPIE.

Kanatani K. *Group-Theoretical Methods in Image Understanding.* Springer Verlag, Berlin, 1990.

Ke Y. and Sukthankar R. PCA-SIFT: A more distinctive representation for local image descriptors. In *Proceedings of CVPR*, pages 506–513, 2004.

Kerautret B., Lachaud J. O., and Naegel B. Comparison of discrete curvature estimators and application to corner detection. In *Proceedings of the 4th International Symposium on Advances in Visual Computing*, ISVC '08, pages 710–719, Berlin, Heidelberg, 2008. Springer-Verlag.

Kiryati N. and Maydan D. Calculating geometric properties from Fourier representation. *Pattern Recognition*, 22(5):469–475, 1989.

Koch M. W. and Kashyap R. L. Using polygons to recognize and locate partially occluded objects. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(4):483–494, 1987.

Koenderink J. J. and Doorn A. J. v. Dynamic shape. Technical report, Department of Medical and Physiological Physics, State University, Utrecht, The Netherlands, 1986.

Krzyzak A., Leung S. Y., and Suen C. Y. Reconstruction of two-dimensional patterns from Fourier descriptors. *Machine Vision and Applications*, 2(3):123–140, 1989.

Lam L., Lee S. W., and Suen C. Y. Thinning methodologies—a comprehensive survey. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14(9):869–885, 1992.

Leu J. G. View-independent shape representation and matching. In *IEEE International Conference on Systems Engineering,* Fairborn, OH, pages 601–604, Piscataway, NJ, 1989. IEEE.

Leymarie F. and Levine M. D. Shape features using curvature morphology. In *Visual Communications and Image Processing IV,* Philadelphia, PA, pages 390–401, Bellingham, WA, 1989. SPIE.

Li B. C. and Shen J. Fast computation of moment invariants. *Pattern Recognition*, 24:807–813, 1991.

Lin C. C. and Chellappa R. Classification of partial 2D shapes using Fourier descriptors. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5):686–690, 1987.

Lindenbaum M. and Bruckstein A. On recursive, o(n) partitioning of a digitized curve into digital straight segments. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15:949–953, 1993.

Loncaric S. and Dhawan A. P. A morphological signature transform for shape description. *Pattern Recognition*, 26:1029–1037, 1993.

Lord E. A. and Wilson C. B. *The Mathematical Description of Shape and Form.* Halsted Press, Chichester, England, 1984.

Loui A. C. P., Venetsanopoulos A. N., and Smith K. C. Two-dimensional shape representation using morphological correlation functions. In *Proceedings of the 1990 International Conference on Acoustics, Speech, and Signal Processing—ICASSP 90,* Albuquerque, NM, pages 2165–2168, Piscataway, NJ, 1990. IEEE.

Lowe D. G. Distinctive image features from scale-invariant keypoints. *International Journal of Computer Vision*, 60(2):91–110, 2004.

Mandelbrot B. B. *The Fractal Geometry of Nature.* Freeman, New York, 1982.

Manzanera A., Bernard T. M., Preteux F., and Longuet B. Ultra-fast skeleton based on isotropic fully parallel algorithm. In *Proc. of Discrete Geometry for Computer Imagery*, 1999.

Maragos P. A. Pattern spectrum and multiscale shape representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11:701–716, 1989.

Maragos P. A. and Schafer R. W. Morphological skeleton representation and coding of binary images. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 34(5):1228–1244, 1986.

Marshall S. Application of image contours to three aspects of image processing; compression, shape recognition and stereopsis. In *Third International Conference on Image Processing and its Applications,* Coventry, England, pages 604–608, Stevenage, England, 1989. IEE, Michael Faraday House.

McCallum D. and Avis D. A linear algorithm for finding the convex hull of a simple polygon. *Information Processing Letters*, 9:201–206, 1979.

McKenzie D. S. and Protheroe S. R. Curve description using the inverse Hough transform. *Pattern Recognition*, 23(3–4):283–290, 1990.

Melkman A. V. On-line construction of the convex hull of a simple polyline. *Information Processing Letters*, 25(1):11–12, 1987.

Miciak M. Radon transformation and principal component analysis method applied in postal address recognition task. *IJCSA*, 7(3):33–44, 2010.

Mundy J. L. and Zisserman A. *Geometric Invariance in Computer Vision.* MIT Press, Cambridge, MA; London, 1992.

Oppenheim A. V., Willsky A. S., and Young I. T. *Signals and Systems.* Prentice-Hall, Englewood Cliffs, NJ, 1983.

Paglieroni D. W. and Jain A. K. Control point transforms for shape representation and measurement. *Computer Vision, Graphics, and Image Processing*, 42(1):87–111, 1988.

Papoulis A. *Probability, Random Variables, and Stochastic Processes.* McGraw-Hill, New York, 3rd edition, 1991.

Pavlidis T. *Structural Pattern Recognition.* Springer Verlag, Berlin, 1977.

Pavlidis T. A review of algorithms for shape analysis. *Computer Graphics and Image Processing*, 7:243–258, 1978.

Pavlidis T. A flexible parallel thinning algorithm. In *Proc. IEEE Computer Soc. Conf. Pattern Recognition, Image Processing*, pages 162–167, 1981.

Pitas I. and Venetsanopoulos A. N. Morphological shape decomposition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(1):38–45, 1990.

Pizer S. M., Oliver W. R., and Bloomberg S. H. Hierarchical shape description via the multiresolution symmetric axis transform. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(4):505–511, 1987.

Pratt W. K. *Digital Image Processing.* Wiley, New York, 2nd edition, 1991.

Quan L., Gros P., and Mohr R. Invariants of a pair of conics revisited. *Image and Vision Computing*, 10(5):319–323, 1992.

Reinhardt J. M. and Higgins W. E. Efficient morphological shape representation. *IEEE Transactions on Image Processing*, 5:89–101, 1996.

Rom H. and Medioni G. Hierarchical decomposition and axial shape description. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15:973–981, 1993.

Rosenfeld A. Digital straight line segments. *IEEE Transactions on Computers*, 23:1264–1269, 1974.

Rosenfeld A. A characterization of parallel thinning algorithms. *Information and Control*, 29: 286–291, 1975.

Rosenfeld A. *Picture Languages—Formal Models for Picture Recognition.* Academic Press, New York, 1979.

Rosenfeld A. and Kak A. C. *Digital Picture Processing.* Academic Press, New York, 2nd edition, 1982.

Rosin P. L. and West G. A. W. Segmentation of edges into lines and arcs. *Image and Vision Computing*, 7(2):109–114, 1989.

Rothwell C. A., Zisserman A., Forsyth D. A., and Mundy J. L. Fast recognition using algebraic invariants. In Mundy J. L. and Zisserman A., editors, *Geometric Invariance in Computer Vision.* MIT Press, Cambridge, MA; London, 1992.

Safaee-Rad R., Benhabib B., Smith K. C., and Ty K. M. Position, rotation, and scale-invariant recognition of 2 dimensional objects using a gradient coding scheme. In *IEEE Pacific RIM Conference on Communications, Computers and Signal Processing,* Victoria, BC, Canada, pages 306–311, Piscataway, NJ, 1989. IEEE.

Samet H. A tutorial on quadtree research. In Rosenfeld A., editor, *Multiresolution Image Processing and Analysis*, pages 212–223. Springer Verlag, Berlin, 1984.

Samet H. Reconstruction of quadtree medial axis transforms. *Computer Vision, Graphics, and Image Processing*, 29:311–328, 1985.

Saund E. Symbolic construction of a 2D scale-space image. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12:817–830, 1990.

Savini M. Moments in image analysis. *Alta Frequenza*, 57(2):145–152, 1988.

Shapiro L. A structural model of shape. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2(2):111–126, 1980.

Shariat H. A model-based method for object recognition. In *IEEE International Conference on Robotics and Automation,* Cincinnati, OH, pages 1846–1851, Los Alamitos, CA, 1990. IEEE.

Shridhar M. and Badreldin A. High accuracy character recognition algorithms using Fourier and topological descriptors. *Pattern Recognition*, 17(5):515–524, 1984.

Sklansky J. Measuring concavity on a rectangular mosaic. *IEEE Transactions on Computers*, 21(12):1355–1364, 1972.

Staib L. H. and Duncan J. S. Boundary finding with parametrically deformable models. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14(11):1061–1075, 1992.

Strackee J. and Nagelkerke N. J. D. On closing the Fourier descriptor presentation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 5(6):660–661, 1983.

Terrades O. R. and Valveny E. Radon transform for lineal symbol representation. In *Proceedings of the 7th International Conference on Document Analysis and Recognition*, pages 195–, Edinburgh, Scotland, August 2003.

Tomek I. Two algorithms for piecewise linear continuous approximation of functions of one variable. *IEEE Transactions on Computers*, 23(4):445–448, 1974.

Toussaint G. A historical note on convex hull finding algorithms. *Pattern Recognition Letters*, 3(1):21–28, 1985.

Tsai W. H. and Yu S. S. Attributed string matching with merging for shape recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 7(4):453–462, 1985.

Turney J. L., Mudge T. N., and Volz R. A. Recognizing partially occluded parts. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 7(4):410–421, 1985.

Tuytelaars T. and Mikolajczyk K. Local invariant descriptors: a survey. *Foundations and Trends in Computer Graphics and Vision*, 3(3):177–280, 2007.

Valera M. and Velastin S. A. Intelligent distributed surveillance systems: A review. In *IEE Proceedings - Vision, Image and Signal Processing*, 2005.

Vernon D. Two-dimensional object recognition using partial contours. *Image and Vision Computing*, 5(1):21–27, 1987.

Wallace T. P. and Wintz P. A. An efficient three-dimensional aircraft recognition algorithm using normalized Fourier descriptors. *Computer Graphics and Image Processing*, 13:99–126, 1980.

Wang D., Haese-Coat V., and Ronsin J. Shape decomposition and representation using a recursive morphological operation. *Pattern Recognition*, 28:1783–1792, 1995.

Weiss I. Projective invariants of shapes. In *Proceedings of the DARPA Image Understanding Workshop,* Cambridge, MA, volume 2, pages 1125–1134. DARPA, 1988.

Weyl H. *The Classical Groups and Their Invariants.* Princeton University Press, Princeton, NJ, 1946.

Wilson R. and Nelson R. *Graph Colourings.* Longman Scientific and Technical; Wiley, Essex, England, and New York, 1990.

Winston P. H., editor. *The Psychology of Computer Vision.* McGraw-Hill, New York, 1975.

Witkin A. P. Scale-space filtering. In Pentland A. P., editor, *From Pixels to Predicates*, pages 5–19. Ablex, Norwood, NJ, 1986.

Woodwark J. *Computing Shape: An Introduction to the Representation of Component and Assembly Geometry for Computer-Aided Engineering.* Butterworths, London–Boston, 1986.

Wright M. W. and Fallside F. Skeletonisation as model-based feature detection. *IEE Proceedings Communication, Speech and Vision*, 140:7–11, 1993.

Wuescher D. M. and Boyer K. L. Robust contour decomposition using a constant curvature criterion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(10):41–51, 1991.

Yuille A. L. and Poggio T. A. Scaling theorems for zero-crossings. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 8(1):15–25, 1986.